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COUETTE FLOW OF A JEFFREY FLUID IN A ROTATING CHANNEL

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ABSTRACT

The steady flow of a Jeffrey fluid between two infinite parallel plates separated by a distance *h* is investigated. The upper plate moves with constant velocity U_0 in $x-$ direction, where the $x-$ axis is taken on the lower stationary plate. The channel is rotating with an angular velocity Ω about y – axis. The velocity field and the temperature distrubation are obtained. When $\lambda_1 \to 0$, the results obtained reduce to corresponding ones of Jana and Datta [21] for Couette flow of a Newtonian fluid in a rotating system. The effects of various physical parameters on the velocity and temperature are discussed in detail. It is observed that the velocity and the temperature increase with an increase in Jeffrey parameter λ ₁.

KEYWORD: Couette Flow; Jeffrey Fluid; Rotating Channel.

INTRODUCTION

Flow of a viscous fluid in a rotating medium is of considerable importance due to the occurrence of various natural phenomena and for its application in various technological situations which are governed by the action of Coriolis force. The broad subjects of oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. The viscous fluid flow problems in rotating medium under different conditions and configurations are investigated by many researchers in the past to analyze various aspects of the problem. Mention may be made of the research studies of Hayat et al. [1], Hayat and Hutter [2] and Das et al. [3]. Such a flow model is of great interest, not only for its theoretical significance, but also for its wide applications to geophysics and engineering. A lot of research work concerning the flow between two parallel plates studied in a rotating system have appeared, for example, Batchelor [4], Ganapathy [5] and Mazumder [6].

Rheological fluids have wide coverage in medicine, engineering and industry. For example, they are important in polymeric and food processes. Further, non-Newtonian electrically conducting fluids in a rotating system are significant in geophysical, cosmical and astrophysical applications. Nalim et al. [7] examined the oscillatory Couette flow mechanical shear loader to simulate in vitro fluid driven shear. Prasad and Kumar [8] implemented a boundary layer assumption for the analysis of MHD oscillatory Couette flow with a porous space. The hydromagnetic Couette flow of viscous fluid in a rotating channel was investigated by Beg et al. [9, 10]. The lower plate of the channel exhibited non-torsional oscillation. Seth et al. [11, 12] and Guria et al. [13] addressed MHD Couette flows in a porous channel and rotating frame. Reddappa et al. [14] investigated Convective Couette flow of a Jeffrey fluid in an inclined channel when the walls are provided with porous lining.

The study of viscous conducting fluids plays a significant role, owing to its practical interest and abundant applications in astro-physical and geo-physical phenomena. The main impetus to the engineering approach to the electromagnetic fluid interaction studies has come from the concept of the hydrodynamics. The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in MHD generators, pumps, accelerators and flow meters and have applications in nuclear reactors, filtration, geothermal systems and others. During the last few decades, interest in mathematical modeling and analysis of flows

involving non-Newtonian fluids in various geometries has been increased. However, there is no model which can lonely predict the behavior of all the non-Newtonian fluids. Among several non-Newtonian models proposed for physiological fluids, Jeffery model is one of the simplest nonlinear non-Newtonian models governing the complex fluid behavior. It is significant because Newtonian fluid model can be deduced from this as a special case by taking the Jeffrey parameter $\lambda_1 = 0$. It has immense importance for their wide applications in engineering industries, for

example in metal extrusion process, wire and blade coating, dying of papers and textiles etc. It is also recognized that many fluids commonly used in industry differ greatly from the Newtonian behavior in their rheology. Using the slip conditions Rudraiah and Wilfred [15] and Vajravelu et al. [16] analyzed the natural convection in an inclined layer bounded by porous material. Chamkha [17] presented analytical solutions for the flow of two-immiscible fluids in porous and non-porous parallel plates. Khan et al. [18] investigated for exact solutions for MHD flow of a generalized Oldroyd fluid with modified Darcy's law. Hayat and Ali [19] investigated the peristaltic motion of a Jeffrey fluid under the effect of a magnetic field. Kothandapani and Srinivas [20] Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel.

The object of this paper is to develop a theoretical model for analyze the Couette flow of a Jeffrey fluid in a rotating channel. The velocity field and the temperature distributions are obtained. When $\lambda_1 \to 0$, the results obtained reduce to corresponding ones of Jana and Datta [21] for Couette flow of a Newtonian fluid in a rotating system. The effects of various physical parameters on the velocity and temperature are discussed in detail. The results are discussed for various physical parameters.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the steady flow of a Jeffrey fluid between two infinite parallel plates separated by a distance *h* is investigated. The upper plate moves with constant velocity U_0 in $x-$ direction, where the $x-$ axis is taken on the lower stationary plate. The channel is rotating with an angular velocity Ω about y – axis. The y-axis normal to the plates and z-axis perpendicular to the *xy-*plane*.* Since the plates are infinite all physical variables will be functions of y only in the steady state. In a rotating frame of reference, the equations of motion are

$$
0 = \frac{\mu}{1 + \lambda_1} \frac{d^2 u}{dy^2} - 2\rho \Omega w \tag{1}
$$

$$
0 = -\frac{\partial p}{\partial y} \tag{2}
$$

$$
0 = \frac{\mu}{1 + \lambda_1} \frac{d^2 w}{dy^2} + 2\rho \Omega u \tag{3}
$$

where (u, o, w) are the velocity components along x, y and z-directions respectively, p is the modified pressure which includes the centrifugal force, μ is the co-efficient of viscosity and ρ is the fluid density, λ_1 is the Jeffrey parameter. The last two terms in equations (1) and (3) are the components of Coriolis force, Ω being the angular

velocity with which the system rotates about the y-axis. Introducing non-dimensional variables

$$
\eta = \frac{y}{h}, \quad u_1 = \frac{u}{U_0}, \quad w_1 = \frac{w}{U_0}, \quad v = \frac{\mu}{\rho}, \quad Da = \frac{k}{h^2}
$$
\n(4)

equations (1) and (3) become

$$
\frac{d^2u_1}{d\eta^2} - 2(1+\lambda_1)a^2w_1 = 0
$$
\n(5)

 \int

 $\overline{}$ ⊱ \mathcal{L}

$$
\frac{d^2 w_1}{d\eta^2} + 2(1 + \lambda_1)a^2 u_1 = 0
$$
\n(6)

where $a^2 = \Omega h^2 / \sqrt{\frac{a^2}{L^2}}$ is the rotation parameter.

The boundary conditions for u_1 and w_1 are

$$
u_1(0) = w_1(0) = 0
$$
, $u_1(1) = 1$ and $w_1(1) = 0$ (7)
Combining equations (5) and (6), we have

$$
\frac{d^2q}{d\eta^2} + 2i(1+\lambda_1)a^2q = 0\tag{8}
$$

where
$$
q = u_1 + w_1
$$
 and $i = \sqrt{-1}$ (9)

The boundary conditions for *q* are

$$
q(0)=0
$$
 and $q(1)=1$ (10)

SOLUTION OF THE PROBLEM

Solving equation (8) subject to boundary conditions (10), we get

$$
q = C_1 e^{(a\sqrt{1+\lambda_1})(1-i)\eta} + C_2 e^{(a\sqrt{1+\lambda_1})(-1+i)\eta}
$$

\nwhere
$$
C_1 = \frac{\sinh(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1}) + i\cosh(a\sqrt{1+\lambda_1})\sin(a\sqrt{1+\lambda_1})}{2[\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})]}
$$

\nand
$$
C_2 = \frac{-[\sinh(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1}) + i\cosh(a\sqrt{1+\lambda_1})\sin(a\sqrt{1+\lambda_1})]}{2[\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})]}
$$

\nOn separating into real and imaginary parts, we get

On separating into real and imaginary parts, we get\n
$$
\begin{bmatrix}\n\vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots\n\end{bmatrix}
$$

$$
u_1 = \left[\frac{1}{\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})}\right] \begin{cases} \sinh(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1})\sinh(a\sqrt{1+\lambda_1})\eta\cos(a\sqrt{1+\lambda_1})\eta \\ + \cosh(a\sqrt{1+\lambda_1})\sin(a\sqrt{1+\lambda_1})\cosh(a\sqrt{1+\lambda_1})\eta\sin(a\sqrt{1+\lambda_1})\eta \end{cases} (12)
$$

$$
w_1 = \left[\frac{1}{\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})}\right] \left[\frac{\cosh(a\sqrt{1+\lambda_1})\sin(a\sqrt{1+\lambda_1})\sinh(a\sqrt{1+\lambda_1})\eta\cos(a\sqrt{1+\lambda_1})\eta\right] - \sinh(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1})\eta\sin(a\sqrt{1+\lambda_1})\eta\right]
$$
\n(13)

SHEAR STRESS

The non-dimensional shear stress due to the primary and secondary flow at the plate η =0 are

$$
\tau_x = \frac{a\sqrt{1+\lambda_1}\left(\sinh(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1}) + \cosh(a\sqrt{1+\lambda_1})\sin(a\sqrt{1+\lambda_1})\right)}{\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})}
$$
(14)

$$
\frac{a\sqrt{1+\lambda_1}\left(\cosh(a\sqrt{1+\lambda_1})\sin(a\sqrt{1+\lambda_1})-\sinh(a\sqrt{1+\lambda_1})\cos(a\sqrt{1+\lambda_1})\right)}{\sinh^2(a\sqrt{1+\lambda_1})+\sin^2(a\sqrt{1+\lambda_1})}
$$
(15)

The resultant shear stress τ at the plate $\eta = 0$ is

$$
\tau = \frac{a\sqrt{2(1+\lambda_1)}}{\sqrt{\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})}}
$$
(16)

HEAT TRANSFER CHARACTERISTICS

The velocity distribution being known, the temperature field can now be determined from the heat transfer equation.

$$
\alpha \frac{d^2 T}{dy^2} + \frac{\mu}{(1 + \lambda_1)\rho c_p} \left[\left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 \right] = 0 \tag{17}
$$

where α is the thermal diffusivity of the fluid, c_p is the specific heat at constant pressure. The last term within the parenthesis is due to the viscous dissipation.

Introducing non-dimensional variables given in (4) and

$$
\theta(\eta) = \frac{T - T_0}{T_1 - T_0}, (T_1 > T > T_0); E = U_0^2 / c_p (T_1 - T_0), \text{Pr} = v / \alpha
$$
\n(18)

Equation (17) becomes

$$
\alpha \frac{d^2 \theta}{d\eta^2} + \frac{\Pr E}{(1 + \lambda_1)} \left[\left(\frac{du_1}{d\eta} \right)^2 + \left(\frac{dw_1}{d\eta} \right)^2 \right] = 0 \tag{19}
$$

The boundary conditions for $\theta(\eta)$ are

$$
\theta(0)=0 \text{ and } \theta(1)=1 \tag{20}
$$

The solution of $\theta(\eta)$ subject to the above boundary conditions is

$$
\theta(\eta) = \eta + \frac{\Pr E}{4(1+\lambda_1)[\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})]} \left[\frac{(\cosh(2a\sqrt{1+\lambda_1}) - \cos(2a\sqrt{1+\lambda_1}))\eta}{-(\cosh(2a\sqrt{1+\lambda_1})\eta - \cos(2a\sqrt{1+\lambda_1})\eta)} \right]
$$
(21)

The rate of heat transfer $\eta = 1$ given by

$$
\frac{d\theta}{d\eta} = 1 - \frac{\Pr E}{4(1+\lambda_1)[\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})]} \left[\frac{(2a\sqrt{1+\lambda_1})(\sinh(2a\sqrt{1+\lambda_1}) + \sin(2a\sqrt{1+\lambda_1}))}{+(\cos(2a\sqrt{1+\lambda_1})\eta - \cosh(2a\sqrt{1+\lambda_1})\eta)} \right]
$$

(22)

It follows from (22) that when $E = E_c$ where

$$
E_c = \frac{4(1+\lambda_1)[\sinh^2(a\sqrt{1+\lambda_1}) + \sin^2(a\sqrt{1+\lambda_1})]}{\Pr\left[(2a\sqrt{1+\lambda_1})(\sinh(2a\sqrt{1+\lambda_1}) + \sin(2a\sqrt{1+\lambda_1})) + (\cos(2a\sqrt{1+\lambda_1})\eta - \cosh(2a\sqrt{1+\lambda_1})\eta)\right]}
$$
(23)

RESULTS AND DISCUSSIONS

The numerical values of primary and secondary velocity are computed from equations (12) and (13) and are depicted in figures 1 to 6 for several values of the rotation parameter a and Jeffrey parameter λ_1 . It is seen from Fig. 1 that the primary velocity decreases with the increase in the rotation parameter *a* . From Fig. 2 we observe that the primary velocity increases near the stationary plate and decreases in the vicinity of the moving plate with an increase in Jeffrey parameter λ_1 with $a = 5$. From Fig.3 it is observed secondary velocity increases for small value of a whereas for large value of a it decreases near the stationary plate and increases in the vicinity of the moving plate. Fig.4 depicts that the secondary velocity increases with increase in Jeffrey parameter λ_1 with $a = 1$. Fig.5 describes that the increase in Jeffrey parameter λ_1 with $a=5$ increases the secondary velocity near the plates and opposite behavior observed in the middle of the channel.

The expression for the temperature is given by equation (21). The temperature profiles are plotted in figures (7) to (10). From Fig.6 we noticed that the increase in a increases the temperature. Fig.7 shows that the increase in λ_1 reduces the temperature. From figures (8) and (9) we observe that the temperature increases with increase in Eckert number (E) and Prandtl number (Pr) .

The values of the resultant shear stress τ for different values of a are given in Table 1($\lambda_1 = 0$, $\lambda_1 = 0.5$). It is found that the resultant shear stress decreases with increase in rotation parameter a for both Newtonian and non-Newtonian fluids. Also observed that τ decreases with increase in λ_1 . The values of rate of heat transfer at η =1 are tabulated in the Table 2($\lambda_1 = 0$, $\lambda_2 = 0.5$) for $E = 0.02$ and for different values of Pr and a. It is found that the rate of heat transfer decreases with increase in either Prandtl number Pr or rotation parameter a for both Newtonian and non-Newtonian fluids. It is also observed that the rate of heat transfer increases with increase in λ_1 . We have calculated the critical Eckert number, given by equation (23), for different values of Pr and a and is given in Table 3($\lambda_1 = 0$, $\lambda_1 = 0.5$). It is found that the critical Eckert number decreases with increase in either Pr or *a* for both Newtonian and non-Newtonian fluids. Further it is observed that the critical Eckert number increases with increase in λ ₁.

Fig. 1. Primary Velocity profiles for different a with $\lambda_{\rm i} = 0.5$

Fig. 3. Secondary Velocity profiles for different a with $\lambda_{\rm i} = 0.5$

Fig. 2. Primary Velocity profiles for different λ_1 with $a = 5$

Fig. 4. Secondary Velocity profiles for different $\,\lambda_{\rm l}^{}$ *with* $a=1$

Fig. 5. Secondary Velocity profiles for different λ_1 *with* $a = 5$

Fig. 7. Temperature profiles for different λ_{\parallel} with $a = 1, E = 0.7, Pr = 0.5$

Fig. 6. Temperature profiles for different a with $\lambda_1 = 1, E = 0.7, Pr = 0.5$

Fig. 8.Temperature profiles for different E with $a = 1, \lambda_1 = 1, Pr = 0.5$

Fig. 9. Temperature profiles for different Pr *with* $a = 1, \lambda_1 = 1, E = 0.7$

	Pr/a	0.5	1.0	2.0	3.0	4.0	5.0	6.0
	0.72	0.9927	0.9916	0.9799	0.9639	0.9496	0.9352	0.9208
$\lambda_1 = 0$ (Newtonian Fluid)	1.00	0.9899	0.9883	0.9720	0.9498	0.9300	0.9100	0.8900
	2.00	0.9798	0.9766	0.9441	0.8996	0.8599	0.8200	0.7800
	3.00	0.9697	0.9649	0.9161	0.8494	0.7899	0.7300	0.6700
	4.00	0.9596	0.9531	0.8882	0.7992	0.7198	0.6401	0.5600
	Pr/a	0.5	1.0	2.0	3.0	4.0	5.0	6.0
$\lambda_{\rm i} = 0.5$								
$(non-$ Newtonian Jeffrey Fluid)	0.72	0.9951	0.9934	0.9816	0.9695	0.9578	0.9460	0.9343
	1.00	0.9932	0.9909	0.9744	0.9576	0.9414	0.9250	0.9087
	2.00	0.9863	0.9818	0.9488	0.9152	0.8827	0.8500	0.8174
	3.00	0.9795	0.9726	0.9232	0.8728	0.8241	0.7750	0.7261
	4.00	0.9727	0.9635	0.8976	0.8304	0.7654	0.7001	0.6347

Table 3: Critical Eckert number Ec for different values of Pr and *a*

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